(Please Write your Exam Roll No. immediately) Exam. Roll No. ……………..

END-TERM EXAMINATION

DECEMBER 2006

Exam Series code: 100163DEC06200116	
Paper Code: MCA-107	**Subject: Discrete Mathematics**

Time: 3 Hours **Maximum Marks: 60**

Note: Attempt question no. 1 and any four of the remaining six questions. Question no. 1 is of 20 marks and the rest are of 10 marks each.

Q. 1. (a) Using Venn diagram prove De-Morgan's rule of set operations.

(b) Define reachability matrix and relate it with transitive closure of a relation on a set. Illustrate your answer with an example.

(c) Prove that there exists infinite many prime numbers.

(d) Show that a regular graph of odd degree cannot have odd number of nodes.

(e) Let f: $G_1 \rightarrow G_2$ be any homomorphism from a group G_1 to G_2 then show that f (e) is identity of G_2 if e is identity of G_1.

(f) Find the coefficient of $w^2 x^5 y^2 z^2$ in $(w + x + y + z)^{15}$.

(g) Draw a Finite state machine M such that M accepts any string in {a, b}* that does not contain two consecutive b's.

(h) Give a regular grammar for the RE a(a + b)* ab.

(i) Draw a lattice for $<P(A), \subseteq>$, where A = {a, b, c}.

(j) Show the equivalence: $\neg(P \leftrightarrow Q) \leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$.

Q. 2. (a) How many friends must you have to guarantee that at least five of them will have birthdays in the same month?

(b) Give a deterministic Finite Automata (FA) with n states, suppose the language accepted by the FA has a string of length more than n. Show that the language has an infinite number of strings.

Q. 3. (a) Use the principle of inclusion and exclusion to find integers between 100 to 10100, both inclusive, which are divisible by 2, 5, 7.

(b) Show that a Group (G,*) is abelian if and only if, $\forall$ a, b ϵ G, we have $a^2 * b^2 = (a * b)^2$.

Q. 4. (a) Define a distributive lattice. Consider the lattice $a<b$, $a<c$, $b<d$, $d<e$, $c<e$. Show that this lattice is not distributive.

(b) Show that a Graph is two colorable if and only if it is a bipartite graph.

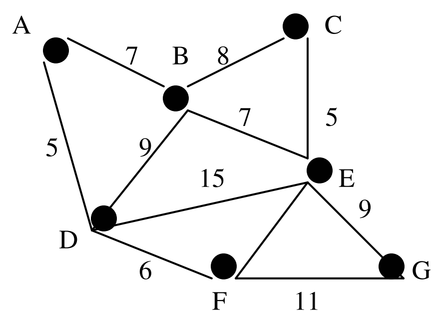
Q. 5. Determine the binomial generating function corresponding to the following discrete numeric function: {0*1, 1*2, 2*3, 3*4, 4*5,…..}. Here * is binary operator of multiplication of real numbers.

(b) Reduce the Boolean function $f(x, y, z) = x^c \wedge (y \vee z) \wedge (x^c \vee y \vee z)$ into CNF.

(c) If v is a cut point of a connected graph G, prove that $\exists$ a partition of a set of node points V- {v} into subsets U and W such that for any points u ϵ U and w ϵ W, the point v is on every u to w path.

Q. 6. (a) Find the explicit formula for $a_n = 2a_{n/2} + 1$; $a_1 = 1$.

(b) Find the minimum spanning tree from the following graph using the Kruskal's algorithm.



Q. 7. (a) Minimize the Boolean expression f(x, y, w, z) = {1, 2, 5, 7, 8, 12, 13, 15} and D = {3, 4, 6}.

(b) Determine the shortest path from A to G in the graph of problem 6(b) above.

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END-TERM EXAMINATION

FIRST SEMESTER [MCA] - DECEMBER 2005

Paper Code: MCA-107	**Subject: Front Discrete Structures**

Time: 3 Hours **(Batch 2004, 2005)** **Maximum Marks: 60**

Note: Attempt five question in all including Q. 1 which is compulsory. Selecting any one question from each unit. Q. 1 carries 20 marks and remaining question carry 10 marks each

Q. 1. Compulsory Question:-

(a) Let A = {1, 2, 3, 4}. Give an example of a relation on A which is both an equivalence relation and a partial order relation.

(b) Find the generating function for the numeric function $a_n = 2^{n+3}$, $n \geq 0$.

(c) Show that

$$\left(\frac{n+1}{r}\right) = \left(\frac{n}{r}\right) + \left(\frac{n}{r-1}\right)$$

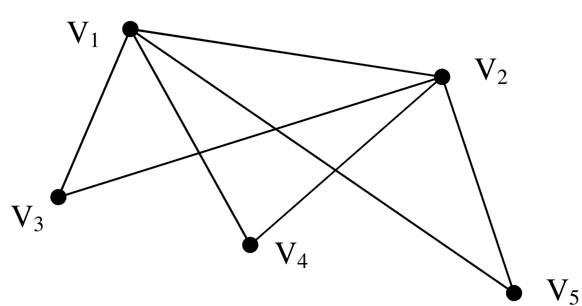
(d) Let D_{30} be the set of all positive divisors of 30. Draw Hasse diagram of lattice D_{30}.

(e) Let a, b, and c be any elements in a Boolean Algebra (B, v, ^, /, O, I). Show that a v (a ^ b) = a.

(f) Let p and q be proportions. Show that $p \rightarrow q \equiv \sim p \vee q$.

(g) Let $f : G \rightarrow H$ be a group homomorphism. If $x \in G$, show that
$(f(x))^{-1} = f(x^{-1})$

(h) Find the adjacency matrix of the graph shown below :-



(i) Define a finite state machine. What are its limitations?

(j) What do you mean by a context free grammar? In what manner it differs from context-sensitive grammar?

UNIT – I

Q. 2. (a) State Pigeonhole Principle. How many people among 2, 00,000 people are born at the same time (hour, minute, seconds)?

(b) Let A = {1, 2, 3, 4} and R = {(1,2), (2,3), (3,4), (2,1)}. Find the transitive closure of R.

Q. 3. (a) Find explicit formula for the sequence defined by $a_1 = 1$, $a_n = 3a_{n-1} + 1$, $n \geq 2$.

(b) Find the corresponding generic function for the generating function

$$\frac{z}{(1 - z)^2}$$

UNIT – II

Q. 4. (a) A software engineer makes the following observations in a computer programming :-

(i) There is an undeclared variable or there is a syntax error in the first five lines.
(ii) If there is a syntax error in the first five lines, then there is a missing semicolon or a variable name is misspelled.
(iii) There is not a missing semicolon.
(iv) There is not a misspelled variable name.

Using propositional calculus, find the mistake in the program.

(b) Using Karnaugh map, find the minimum sum-of-product form for
$E = xy' + xyz + x'y'z' + x'yz\,t'$

Q. 5. (a) Prove that every chain is a distributive lattice.

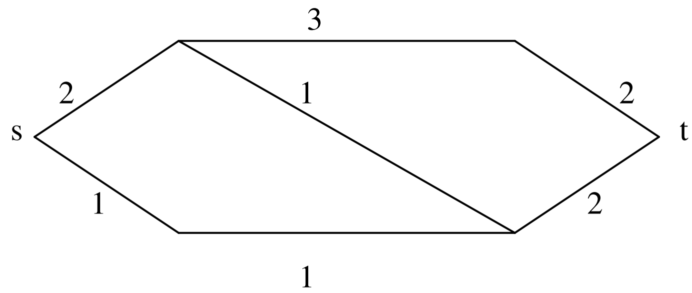
(b) Design a three-input-minimal AND-OR circuit with the following truth table:-
T = [A, B, C, L] = [00001111, 00110011, 01010101, 11001101]

UNIT – III

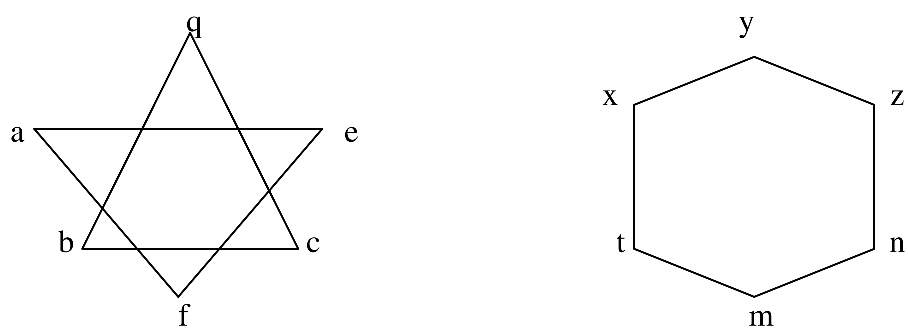
Q. 6. (a) Giving graphical representation discuss Seven Bridges Problem. Was it possible for a citizen to make a tour of the city and cross each bridge exactly twice? Give reasons.

(b) Define a cyclic group. Show that any two cyclic groups of the same order are isomorphic.

Q. 7. (a) Write Dijkstra's Shortest Path Algorithm and grow a Dijkstra tree from s to t in the graph given below :-



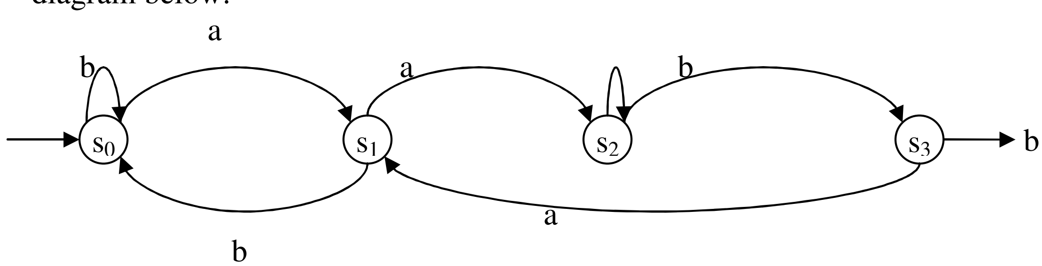
(b) (i) Are the following graphs isomorphic? Give reasons.



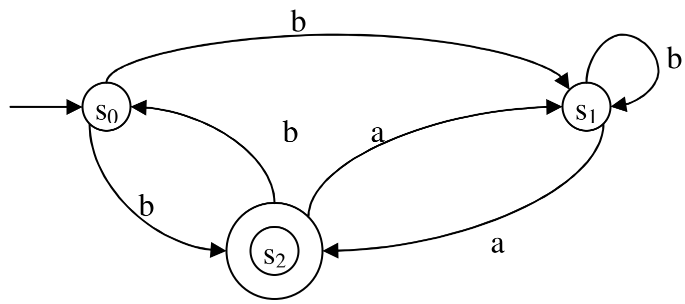
(ii) Is there a non-empty simple graph with twice as many edges as vertices? Give arguments.
(iii) Give an example of a graph that has an Hamiltonian circuit but nit an Euler circuit.
(iv) What are the generators of additive group of integers?
(v) Define even and odd punctuations. What is the order of the set A_n of all even permutations in symmetric group S_n ?

UNIT – IV

Q. 8. (a) Find the language accepted by the automation M shown in the transition diagram below:-



(b) Determine whether the string abbaa accepted by the finite state automation having transition diagram given below :-



Q. 9. (a) Find a context-free grammar G which generates the language

$$L = \{a^n b^n : n > 0\}$$

(b) State and prove Pumping Lemma

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END-TERM EXAMINATION

FIRST SEMESTER [MCA] - DECEMBER 2004

Paper Code: MCA-107	**Subject: Front Discrete Structures**
Time: 3 Hours	**Maximum Marks: 60**

Note: Attempt five question in all including Q. 1 which is compulsory.

Q. 1. Answer the following: **20**

(a) Let A = {1, 2, 3} and {2, 3, 5. Which of the following are relations from A to B?

(i){(1,3),(2,2), (3,5),(1,5)}

(ii) {(2,2),(2,1), (2,3),(2,5)}

(b) Prove that n! (n+2) = n! + (n+1)!.

(c) Verify the equality 2C(7,4) = C(8,4)

(d) If the inverse if a is a^{-1} , then the inverse of a^{-1} is a i.e. $(a^{-1})^{-1}$ =a.

(e) Let A{1, 7, 11, 13, 77, 91, 143, 1001} be ordered by the relation "x divides y". Draw its diagram.

(f) Find the duals of the following parts :-

(i) ({10, 1, 2, 3}, $\leq$)

(ii) (N, 1)

(g) How many edges are there in a graph with 12 vertices each of degree 6?

(h) Suppose that a graph has 7 vertices. Each of degree 6. Into how many regions in the plane divided by planar representation of this graph?

(i) Let K = {a, ab, a^2} and L { b^2 , ab, a} be a language over A={a, b}. Find

(i) KL (ii) LL

(j) Consider the Language L= {ab, C} over A = {a, b, c} find

(i) L^0 (ii) L^3

Q. 2. (a) For any relation R in a set A, prove that **5**

(i) R is symmetric if and only if R^{-1} is symmetric.

(ii) R is reflexive if and only if R^{-1} is reflexive.

(b) Use mathematical induction to show that 5 divides n^5 –n whenever n is a non negative integer. **5**

Q. 3. (a) If C(n,a): C(n, r + 1)=1:2 and C(n, r + 2) = 2:3 determine values of n and r. **3**

(b) For the post of 5 teachers, there are 23 applications, 2 posts are reserved for SC candidates and there are 7 SC candidates among the applicants in how many ways the section is made. **3**

(c) A gentleman invites a party of (m+n) friends to a dinner and place m at one table and the remaining n at another, the tables being round. Prove that the number of ways in which he can arrange them among themselves is (m+n)!/mn. **4**

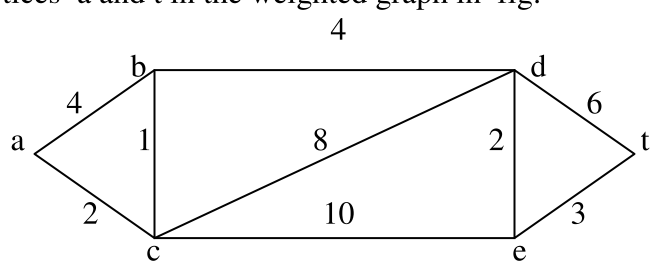
Q. 4. (a) Obtain the equivalent conjutive normal formal or product of sum canonical form of Boolean expression in three variable x_1, x_2, x_3

(i) x_1, x_2

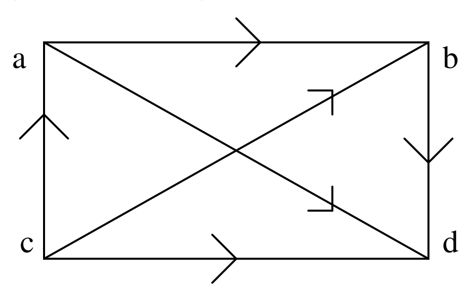
(ii) $x_1 . (x_2^1 . x_3^1)$ **5**

(b) A detective has interviewed four witnesses to a crime from the stories of the witness the detective has concluded that if the butler is telling the truth then so is the cook, the cook and the gardener cannot both be telling the truth ; the gardener and the handyman are not both lying ; and if the handyman is telling the truth then the cook is lying. For each of the four witnesses, can the detective determine whether that person is telling the truth or lying? Explain in your reasoning. **5**

Q. 5. (a) Use Dijkstra's Algorithm to find the length of the shortest path between the vertices a and t in the weighted graph in fig: **5**



(b) Let A = {1, 2, 3, 4, 5, 6}. Let R be the relation in A defined by xRy if x divides y. **5**



(i) Write R as a set of ordered pairs.

(ii) Draw a diagram of the directed graph which corresponds to R.

Q. 6. (a) Let A = {a, b}. Describe verbally the following language over A **3**

(i) $L_1 = \{a \,.\, b)^m : m > 0\}$

(ii) $L_2 = \{a^r\, b\, a^s\, b\, a^t : r, s, t \geq 0\}$

(iii) $L_3 = \{a^2\, b^m a^3 : m > 0\}$

(b) Suppose $\alpha = \alpha$ (W) for some input W and suppose $\alpha \rightarrow \beta \rightarrow \alpha$. Can M recognize W? **3**

(c) Let A= {a, b}. Find a Turing machine M which recognizes the Language $L = \{ab^n : n \geq 0\}$, that is, where L consists of all the words W beginning with one a and followed by one or more b's. **4**
