# END TERM EXAMINATION

Paper code: MCA105 Subject: Discrete mathematics

Note: Attempt any five questions including Q no. 1 which is compulsory. Select one question each from a unit.

Q1 (a) Show that $A \cap B = A \cup B$. where A and B are sets.

(b) Define binary relations. How many binary relations are there on a set A with n elements.

(c) find the minimum number of students in a class so that three of them are born the same day.

(d) how many ways can a group of 6 people be seated around a table?

(e) Let $D_{105}$ be the set of all divisors of 105. Draw a hasse diagram of lattice $D_{105}$ .

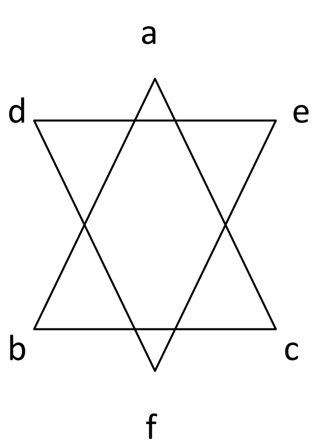
(f) Show that $D_{20}$ is not a finite Boolean algebra with the partial order of divisibility.

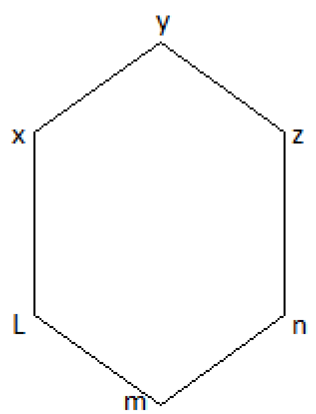
(g) What are applications of number theory in computer science?

(h) Let (G,.) be a group. Prove that $(xy)^{-1} = y^{-1}x^{-1}$ .

(i) Giving graphical representation discuss seven bridges problem. Was it possible for a citizen to make a tour of the city and across each bridge exactly twice? Give reasons.

(j) Are the following graphs isomorphism? Give reasons.






**(2*10=20)**

# UNIT-1

Q2 (a) Let R be a relation on the set of real numbers such that aRb iff a-b is an integer. Show that R is an equivalence relation.

(b) what do you mean by indirect proof? Using indirect proof prove that "if 3n+2 is odd, then n is odd".

(c) show by mathematical induction $\forall n \in N \left[\sum^{n}_{i=0} i = n(n+1)/2\right]$

**{3+3+4}**

Q3 (a) Without using truth table, prove the following

$(\neg p \vee p) \wedge (p \wedge (p \wedge q)) \equiv (p \wedge q)$

(b) Let be the set of all strings of length 3 made up of 0's and 1's i.e.

$\sum 3$={000,001,010,011,100,101,110,111}. h: P({a,b,c})$\rightarrow$ $\sum$ 3,h(X)=$s_1 s_2 s_3$

With $s_1$ if a$\in$X and 0 otherwise $s_2$=1 if b$\in$X and 0 otherwise, $s_3$=1 if c$\in$X and 0 otherwise. Show that h is a bijection. **(5+5)**

# UNIT-2

Q4 (a) Let (L, $\leq$) be a bounded distributive lattice with 1 and 0 as unit and zero elements of L respectively. (i) Prove the Demorgan's Law. (ii) Show that if the complement of an element in L exists then it is unique.

(b) Minimize the Boolean expression F=A'C+AB'C+A'B+BC. **(5+5)**

Q5 (a) If lattices ($L_1$, $\leq$ ) and ($L_2$ , $\leq$) are lattices, show that ($L_1$X$L_2$, $\leq$) is also a lattice.

(b) Minimize the Boolean expression f(x,y,z,w)= $\sum$ (0,3,4,5,7) and d(x,y,z,w)= $\sum$ (8,9,10,11,12,13,14,15) **(5+5)**

# UNIT-3

Q6 (a) State and prove Euclid's division algorithm.

(b) Explain Euclidean algorithm to find the gcd of two nos. by taking example.

(c) State and prove Fermat's Little theorem. **(3+4+3)**

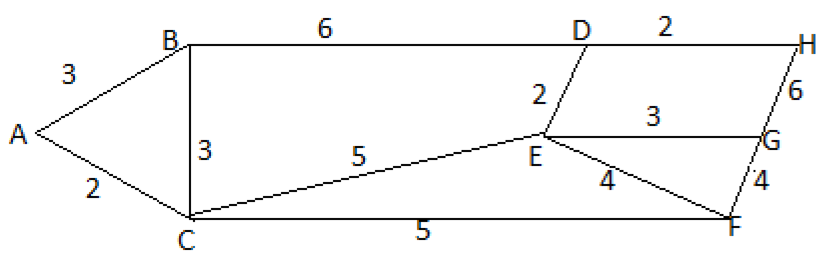
Q7 (a) Show that in a subset H of a group (G,*) if $a*b^{-1}$ is in H for all a, b in H, then H is a subgroup of G.

(b) Let (G,.) be a group. Let (H,.) be a subgroup of (G,.). Show that G=H$\cup$Ha$\cup$Hb………..

where a,b$\in$G. **(5+5)**

# UNIT-4

Q8 (a) Using Prim's algorithm, find minimal spanning tree from the following graph.

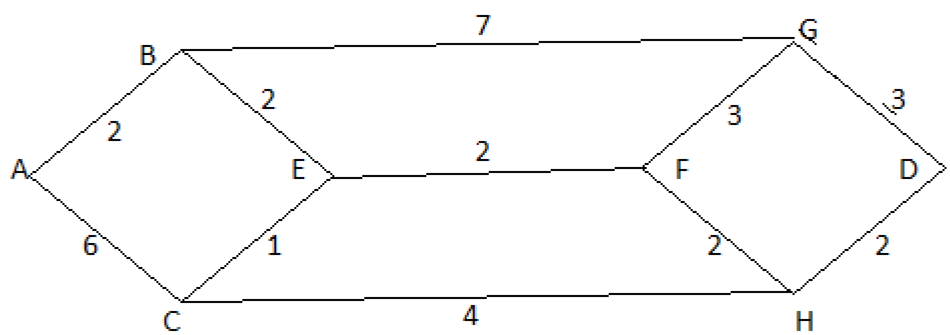


(b) Let G=(V,E) be an undirected graph with edges with then show that 2e= $\sum$deg(v)

v$\in$V.

Symbols have own meaning. What conclusions can you draw from this result? **(7+3)**

Q9 (a) Write steps for finding the shortest path between two vertices of a graph using Dijsktra's method. Hence find the shortest path between node A and D.




(b) Show that a graph is two colorable if and only if it is a bipartite graph. **(7+3)**