(Please write you Exam Roll No.) Exam Roll No. ………………..

END TERM EXAMINATION

FIRST SEMESTER [MCA] JANUARY 2011

Paper Code: MCA105	**Subject: Discrete Mathematics (Batch: 2010)**
Time: 3 Hours	**Maximum Marks: 60**

Note: Q. no. 1 is compulsory. Attempt one question from each unit.

Q1 (a) Construct the truth table for $(p \wedge q) \rightarrow (p \wedge q)$.

(b) Write the PDNF (Principal Disjunctive Normal Form) of $(p \wedge \sim q)$.

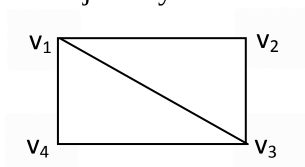
(c) Prove that $(A - C) \cap (C - B) = \emptyset$ analytically, where A, B, C are sets.

(d) Define binary relation from one set to another. Give an example.

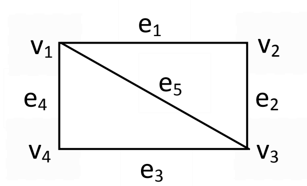
(e) If $*$ is the binary operation on the set R of real number defined by $a * b = a + b + 2ab$, a, b $\in$ R. Find a^{-1}.

(f) If the permutations of the elements of {1, 2, 3, 4, 5} are given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$ then find $\alpha\beta$.

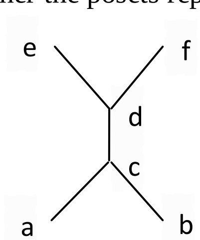
(g) Write down the adjacency matrix of the following graph



(h) Find the incidence matrix of the graph given below



(i) Determine whether the posets represented by the Hasse diagram are lattices.



(j) In Boolean Algebra if a + b = 1 and a . b = 0, show that the complement of every element 'a' is unique. **(2x10=20)**

UNIT – I

Q2 (a) Prove that $\forall x(P(x) \rightarrow (Q(y) \wedge R(x))), \exists x P(x) \Longrightarrow Q(y) \wedge \exists x(P(x) \wedge R(x))$. **(5)**

(b) Without constructing the truth tables, find the principal disjunctive normal forms of the following statements $P \wedge \sim(Q \wedge R) \vee (P \rightarrow Q)$. **(5)**

Q3 (a) If R is the relation on the set of integers such that $(a, b) \in R$, if and only if $3a + 4b = 7n$ for some integer n, prove that R is an equivalence relation. **(5)**

(b) Determine the number of integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 32$, where $x_i \geq 0$, $1 \leq i \leq 4$. **(5)**

UNIT – II

Q4 (a) Prove that $D_{42} \equiv \{S_{42}, D\}$ is a complemented lattice by finding the complements of all the elements, where S_{42} is the set of all divisions of the positive integer 42 and D is relation of Division. **(5)**

(b) In Boolean Algebra, show that $(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a)$. **(5)**

Q5 (a) Find the minimum of the following Boolean function $f(a, b, c) = \sum(0, 1, 2, 3, 5, 7)$. **(5)**

(b) Solve the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n + 1)2^n$. **(5)**

UNIT – III

Q6 (a) Show that every group of order 3 is cyclic and every group of order 4 is obelian. **(5)**

(b) If H is a subgroup of G such that $x^2 \in H$ for every $x \in G$, prove that H is a normal subgroup of G. **(5)**

Q7 (a) Show that every quotient group of a cyclic group is cyclic. **(5)**

(b) Find the code words generated by the encoding function $e: B^2 \longrightarrow B^5$ with respect to the parity check matrix $H = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. **(5)**

UNIT - IV

Q8 (a) Prove that the number of edges in a bipartite graph with n vertices is at most $\left(\frac{n^2}{2}\right)$. **(5)**

(b) The incidence matrix of two pairs of graphs are as follows

$I_G = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix}$, $I_H = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{vmatrix}$. Examine the isomorphism of G and H graphically o by finding a permutation matrix. **(5)**

Q9 (a) Give an example of a graph which contains- **(5)**

(i) an Eulerian circuit that is also a Hamiltonian circuit.

(ii) a Hamiltonian circuit, but not an Eulerian circuit.

(b) Construct a binary tree whose in order and pre order transversal are respectively EACIFHDBG and FAEICDHGB. **(5)**

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